

Analysis of Finite-Differencing Errors to Determine Cell Size When Modeling Ferrites and other Lossy Electric and Magnetic Materials Using FDTD

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Abstract

The focus of this paper is the modeling of materials that have both significant electric and magnetic losses, such as ferrites, using FDTD. The primary contribution is identifying appropriate cell sizes when modeling these types of materials. It is shown that finite-differencing errors increase in lossy media compared to lossless media when sampling at the same number of cells per wavelength. Losses in a medium are defined by the ratio of the attenuation constant, α , to the phase constant, β , since that ratio accounts for all losses, whether they be electric or magnetic. In addition to a detailed finite-differencing error analysis, a simple approximation is given for selecting a cell size in a lossy material that will give the same finite-differencing error as ten cells per wavelength in a lossless material.

This paper also presents a means for deriving pure real constitutive parameters from complex constitutive parameters. Being able to make such calculations is useful in cases where complex constitutive parameters are given for a material, and the FDTD model being used only accepts pure real constitutive parameters, as is the case for several contemporary models. Comparisons of theoretical and FDTD-modeled reflection and transmission show that the derived, real constitutive parameters are valid.

Introduction

Ferrites [1] are a commonly used material for suppressing unwanted radiation from wires, integrated circuits, equipment enclosures and anechoic chambers. The electromagnetic properties of ferrites are such that fields can penetrate into them and are dissipated through both electric and magnetic loss mechanisms. Some of the properties that make ferrites desirable from a shielding perspective also make them challenging to model using time-domain approaches such as FDTD. One of the challenges is in choosing an appropriate cell size, as the exponentially-decaying field behavior introduces finite-differencing errors that are not accounted for in commonly-accepted, cells-per-wavelength rules. In particular, it is shown that there is a greater finite-differencing error associated with lossy materials compared with lossless materials when both are modeled with the same number of cells per wavelength.

Other challenges associated with the modeling of ferrites are that their electromagnetic properties are anisotropic, nonlinear, and frequency dependent. These properties have been accounted for in FDTD using several different approaches. One relates the flux density vector to the magnetic field vector within the medium using a frequency-dependent permeability tensor. This relationship is converted into the time domain by performing a convolution at every FDTD time step.

Another type of ferrite treatment in FDTD is based on Gilbert's equation of motion that describes the interaction between the magnetic field and the magnetization vector in the time domain [2, 3, 4, 5, 6, 7]. The equation of motion is solved consistent with Maxwell's equations.

The results presented in this paper for selecting cell size are applicable for the standard FDTD algorithm as well as for the models that account for the ferrite electromagnetic properties described above. When applying the standard FDTD algorithm, ferrite electromagnetic characteristics at a single frequency are used, and those characteristics in conjunction with the highest frequency of interest are used to determine cell size following the procedure given here. If a broadband model is used, cell sizes for all frequencies within the spectral range of interest can be calculated, and the smallest cell size in that range should be selected for use in the FDTD model to achieve the desired error.

This paper begins by discussing electric and magnetic losses, along with an appropriate method for representing them in materials where both loss mechanisms are present. Following that is an intuitive explanation as to why lossy materials require more cells per wavelength than lossless materials to realize the same finite-differencing error. Then, an analysis is given that quantifies finite-differencing error as a function of medium loss and cell size. Finally, a method for calculating pure real constitutive parameters from complex constitutive parameters is provided along with validation data. This last development is given to enable the modeling of complex materials using an FDTD algorithm that only accepts real constitutive parameters.

Accounting for Electric and Magnetic Loss Mechanisms

There are three basic loss mechanisms for electromagnetic fields. The first mechanism is caused by the flow of electric charges through a medium, and it will occur whenever there is finite conductivity. The average dissipated power density associated with this type of loss is $\frac{1}{2}\sigma|E|^2$, where σ is the conductivity and E is the electric field. A second loss mechanism, also associated with the electric field, is caused by the movement of electric dipoles when an alternating field is applied. It is represented mathematically by a complex permittivity $\epsilon(\omega) = \epsilon'(\omega) - j\epsilon''(\omega)$ which is generally approximated using the equation describing the motion of electric dipoles [8]. The average dissipated power density associated with this loss is proportional to the imaginary part of the complex permittivity and is given by $\frac{1}{2}\omega\epsilon''|E|^2$.

A third type of loss, associated with the magnetic field occurs as the result of magnetic dipole motion and is the dual of electric dipole loss [9]. Accordingly, its mathematical representation as a complex permeability is in the same form as the electric field loss mechanism: $\mu(\omega) = \mu'(\omega) - j\mu''(\omega)$. The corresponding dissipated power density is expressed as $\frac{1}{2}\omega\mu''|H|^2$, where H is the magnetic field. All three of these loss mechanisms are functions of frequency, and hence will introduce additional frequency dispersion.

Having complex constitutive parameters will introduce additional terms in Maxwell's equations (Faraday's and Ampere's law), and are expressed conveniently in frequency domain as:

$$\nabla \times E = -j\omega\mu H = -j\omega(\mu' - j\mu'')H = -\omega\mu''H - j\omega\mu'H \quad (1)$$

$$\nabla \times H = \sigma E + j\omega\varepsilon E = \sigma E + j\omega(\varepsilon' - j\varepsilon'')E = (\sigma + \omega\varepsilon'')E + j\omega\varepsilon'E \quad (2)$$

The imaginary term on the right side of Equation 1, $-j\omega\mu'H$, represents the magnetic displacement current density, while the real part, $-\omega\mu''H$, represents the magnetic conduction current density. Consistent with what is stated above it is the magnetic conduction current density that relates to the magnetic loss, and it is proportional to μ'' .

Similarly, the imaginary term on the right side of Equation 2, $j\omega\varepsilon'E$, represents the electric displacement current density, while the real part, $(\sigma + \omega\varepsilon''E)$, represents the electric conduction current density. As with the magnetic field, it is the electric conduction current density that relates to the electric loss, and it is a function of both ε'' and σ . It is common practice to include the static conductivity, σ , into the imaginary part of the complex permittivity. Specifically:

$$\varepsilon'' = \frac{\sigma}{\omega} + \varepsilon_a'' \quad (3)$$

where ε_a'' is the imaginary part of complex permittivity which represents only the losses caused by electric dipole motion. Static conductivity is incorporated into the imaginary component of the complex permittivity for all of the data shown in this paper, including the manufacturer provided data referenced here.

An example of complex permittivity and permeability as a function of frequency for a ferrite material is shown in Figure 1, and the data were provided by the manufacturer on request. The company's brochure for the material contained transmission and reflection data, but not the detailed information shown in Figure 1; this particular material is used for reducing reflections in anechoic chambers. While different ferrite materials will exhibit different characteristics, the type of frequency dependence is typical.

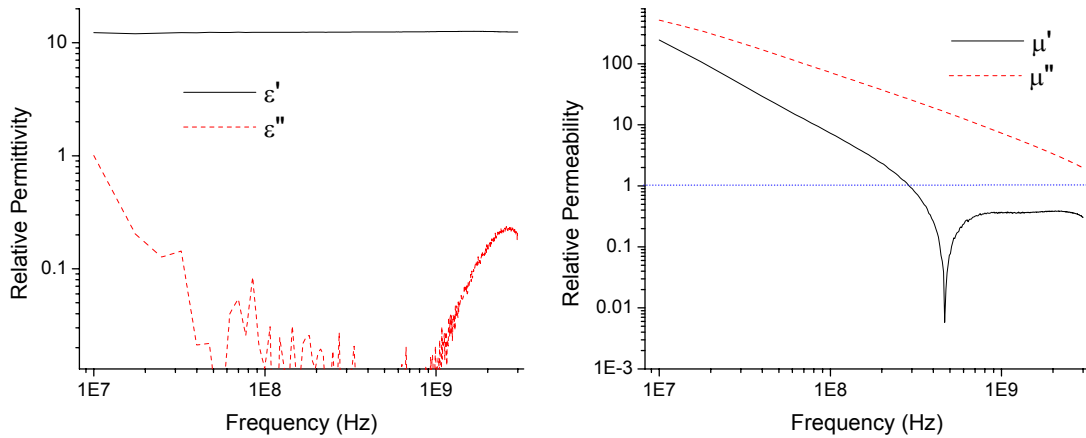


Figure 1 Complex Permittivity and Permeability of Material42 as a Function of Frequency

The complex constitutive parameters of a medium such as the ones plotted in Figure 1 are commonly determined by measuring the reflection and transmission characteristics of a

coaxial transmission line terminated with a sample of that medium. This measurement provides the four known variables (the complex transmission and reflection coefficients at a particular frequency) needed to solve for the four unknowns (the complex permittivity and permeability).

Electromagnetic loss is typically expressed in terms of electric and magnetic loss tangents: $\tan(\delta_e) = \frac{\epsilon''}{\epsilon'}$ and $\tan(\delta_m) = \frac{\mu''}{\mu'}$. However, because a composite loss tangent is not defined for materials that have both significant electric and magnetic losses, another measure of loss is used here. Specifically, losses in a medium are defined here by the ratio of the attenuation constant, α , to the phase constant, β , since that ratio accounts for all losses, whether they be electric or magnetic. That ratio indicates how quickly a sinusoidal signal decays compared to its spatial phase variation, which can be related directly to finite-differencing error as is shown below.

Relationship between Losses and Cell Size: An Intuitive Explanation

To illustrate why a smaller cell size is needed when modeling lossy materials, consider Figure 2, which plots a sine wave propagating in a lossless medium. The accompanying spatial frequency spectrum in that figure reveals the expected responses for a sinusoidal excitation. In this case, the spatial bandwidth is clearly defined, relating only to the temporal frequency of the wave and the velocity of propagation in the medium. Further, a sample size referenced to a wavelength, such as 10 cells per wavelength, will result in the same finite-differencing error regardless of media type or frequency.

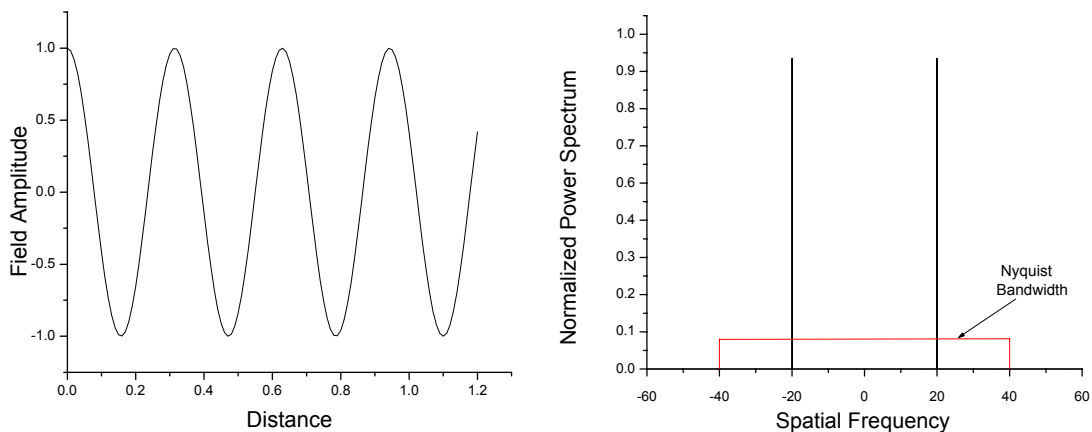


Figure 2 A Propagating Sinusoid and Its Spatial Frequency Spectrum In a Lossless Medium

Now consider Figure 3, which plots a sinusoidal waveform propagating in a lossy medium. In this case, the exponential attenuation of the waveform in the spatial domain results in an increased bandwidth in the spatial-frequency domain. Because of the inverse relationship between bandwidth and sample size, this increased bandwidth necessitates a smaller spatial sample size to achieve the same degree of finite-differencing accuracy as

would be realized for a wave of the same frequency traveling at the same speed, but in a lossless medium.

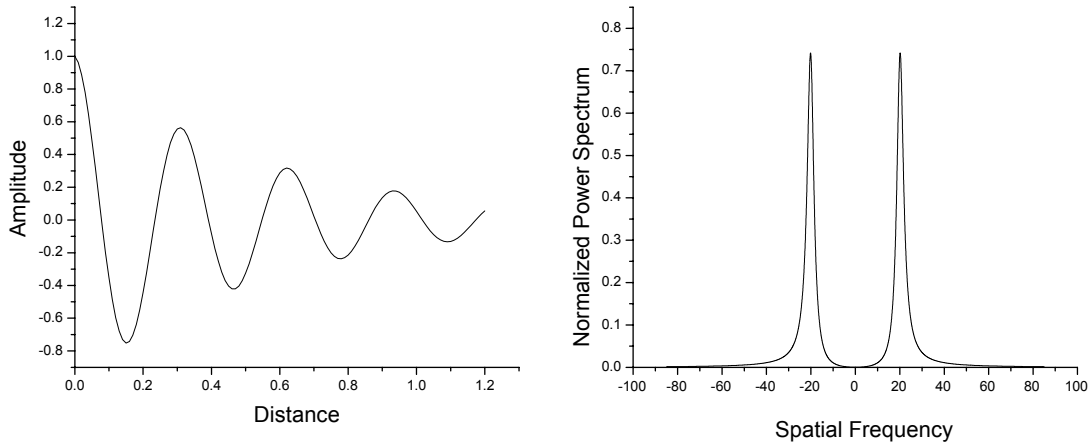


Figure 3 A Propagating Sinusoid and Its Spatial Frequency Spectrum In a Lossy Medium

While the spatial spectral analysis shows that cell size needs to be decreased in a lossy medium, it does not provide a clear means for assessing errors when performing finite differencing. Consequently, another approach, presented in the following section, quantitatively relates cell size to finite differencing errors.

Relationship between Losses and Cell Size: A Quantitative Assessment Based on the Analysis of Finite-Differencing Errors

Although there are other sources of error when using FDTD, the most significant error for most applications results from the use of finite differencing to approximate the temporal and spatial first derivatives in Maxwell's curl equations [10]. Consequently, the error analysis performed here focuses on errors associated with that approximation. The approach used here to identify an appropriate cell size for any material is to calculate finite-differencing error for a sinusoidal excitation, and reference that error to the error that would exist for a lossless medium sampled at 10 cells per wavelength [11].

For an arbitrary function, $y(x)$, the spatial derivative approximation at x_0 using finite differencing is defined as:

$$\left. \frac{dy(x)}{dx} \right|_{x=x_0} \approx \frac{y(x_0 + \Delta x/2) - y(x_0 - \Delta x/2)}{\Delta x} \quad (4)$$

Assuming a sinusoidal excitation of unity amplitude propagating in a lossless medium, $y(x) = \cos(\beta x)$, the error in the finite-difference approximation as a function of distance along the waveform is given by:

$$|\Delta y'(x)| = \left| \frac{d \cos(\beta x)}{dx} - \frac{\cos\left(\beta\left(x + \frac{\Delta x}{2}\right)\right) - \cos\left(\beta\left(x - \frac{\Delta x}{2}\right)\right)}{\Delta x} \right| \quad (5)$$

An algebraic manipulation of Equation 5 puts it in a more usable form:

$$|\Delta y'(x)| = \left| \beta \left(1 - \frac{\sin\left(\beta \frac{\Delta x}{2}\right)}{\beta \frac{\Delta x}{2}} \right) \sin(\beta x) \right| \quad (6)$$

From Equation 6, it is seen that the maximum error for a cosine will occur when $\beta x = 90^\circ$, which is an expected result, since that is where the slope is steepest. Because it is the maximum error within an interval that is of interest here, the $\sin(\beta x)$ term in Equation 6 is omitted in subsequent calculations of absolute error for lossless media.

Although not directly relevant to the analysis given here, it is interesting to note that the relative finite-differencing error as a function of cell size can be readily calculated. For example, the relative error in a lossless medium corresponding to 10 cells per wavelength is:

$$\frac{|\Delta y'(x)|}{|y'(x)|} = \left| \frac{\Delta y'(x)}{-\beta \sin(\beta x)} \right| = \left| 1 - \frac{\sin\left(\frac{\pi}{10}\right)}{\frac{\pi}{10}} \right| = 0.0164 = 1.64\%. \quad (7)$$

Because the relative error becomes singular in lossy media (the derivative goes to zero in places where the approximated derivative does not), it is the absolute error that is used in error analysis presented here.

A sinusoidal field propagating through a lossy medium will attenuate exponentially as $e^{-\alpha x} \cos(\beta x)$, and the absolute maximum error in the finite-difference approximation to its derivative is given by:

$$|\Delta y'(x)| = \left| \frac{d\left(e^{-\alpha x} \cos(\beta x)\right)}{dx} - \frac{e^{-\alpha\left(x + \frac{\Delta x}{2}\right)} \cos\left(\beta\left(x + \frac{\Delta x}{2}\right)\right) - e^{-\alpha\left(x - \frac{\Delta x}{2}\right)} \cos\left(\beta\left(x - \frac{\Delta x}{2}\right)\right)}{\Delta x} \right| \quad (8)$$

Rearranging this expression, and normalizing this error to compensate for the exponential decay, Equation 8 can be written as:

$$|\Delta y'| = \sqrt{\left(\alpha - \frac{2 \sinh\left(\alpha \frac{\Delta x}{2}\right) \cos\left(\beta \frac{\Delta x}{2}\right)}{\Delta x} \right)^2 + \left(\beta - \frac{2 \cosh\left(\alpha \frac{\Delta x}{2}\right) \sin\left(\beta \frac{\Delta x}{2}\right)}{\Delta x} \right)^2} \quad (9)$$

Letting the variable k be the number of cells per wavelength (i.e., $k = \lambda/\Delta x$), Equation 9 becomes:

$$|\Delta y'| = \beta \sqrt{\left(\frac{\alpha}{\beta} - \frac{2 \sinh\left(\frac{\alpha \pi}{\beta k}\right) \cos\left(\frac{\pi}{k}\right)}{\frac{2\pi}{k}} \right)^2 + \left(1 - \frac{2 \cosh\left(\frac{\alpha \pi}{\beta k}\right) \sin\left(\frac{\pi}{k}\right)}{\frac{2\pi}{k}} \right)^2} \quad (10)$$

In this form, Equation 10 provides the absolute finite-differencing error in terms of α , β and the number of cells per wavelength. To illustrate the effect of loss on finite-differencing error, refer to Figure 4, which plots relative error as a function of cell size for five different α/β values. Relative error is referenced to the error that would occur in a lossless medium sampled at ten cells per wavelength. Accordingly, the error curve for the lossless case ($\alpha/\beta = 0$) has a value of one when $\Delta x = 0.1\lambda$. The relative error values plotted in Figure 4 were obtained by calculating the absolute error using Equation 10, and dividing it by the error value for the lossless case given in Equation 6. Figure 4 reveals the expected result that finite-differencing error will increase with either cell size or medium loss. It should be noted that the wavelength referenced here is the wavelength in the medium.

The significance of the error versus cell size data presented here is that it provides a means for achieving a desired finite-differencing accuracy when modeling lossy media

with FDTD.

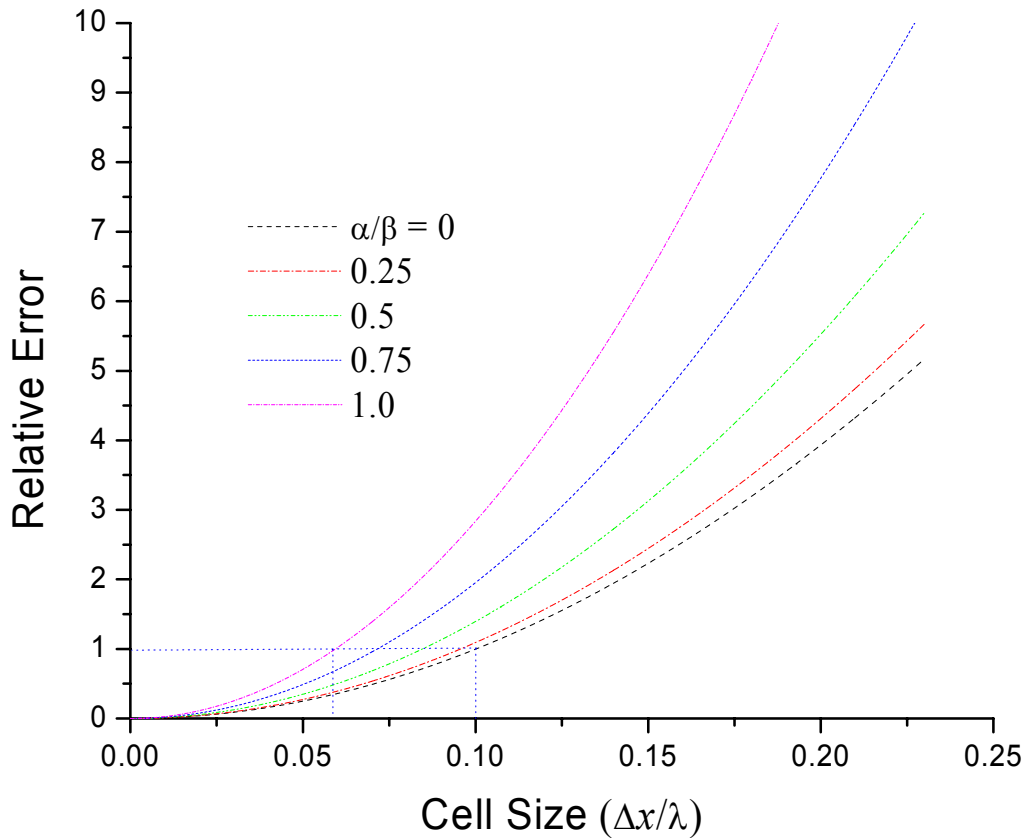


Figure 4 Relative Finite-Differencing Error as a Function of Cell Size and α/β

To provide perspective on how the information in Figure 4 might be applied in practice, Figure 5 plots the loss factor, α/β , as a function of frequency for several different ferrite types, where the labels indicate the type of application for which a ferrite is used. It should be noted that the constitutive parameters for the material labeled “Beads & Chokes 1” are plotted in Figure 1. The data of Figure 4 can be used to calculate a cell size at a particular frequency to achieve a desired finite-differencing error. The error associated with a particular cell size in wavelengths is determined from Equation 10 above, and the wavelength in the medium can be determined from the constitutive parameters at the frequency of interest. However, for most modeling applications involving ferrites, it is may not be necessary to perform the detailed calculations, because for the majority of conditions, α/β is close to one (recall that in a good conductor, $\alpha \approx \beta$). Consequently, a rule of thumb is offered here, named “Dragan’s Rule” after one of the authors, that provides a readily-applied approach to determining cell size for modeling ferrites and other high-loss materials. Dragan’s rule is that using twenty cells per wavelength in a high-loss medium will give approximately the same finite-differencing error as ten cells per wavelength in a lossless medium. The justification for this rule is seen in Figure 4, where the finite-differencing error for a medium with $\alpha/\beta = 1$ is the same as for a lossless me-

dium sampled at ten cells per wavelength if the number of cells per wavelength is roughly doubled for the lossy medium.

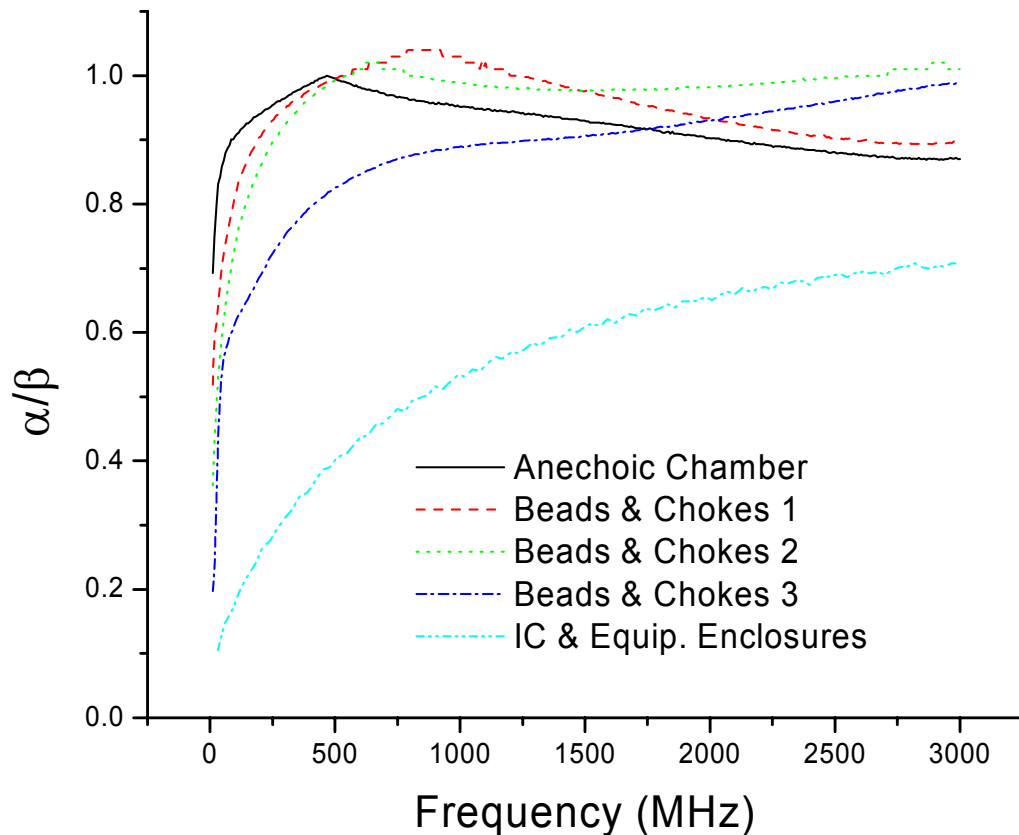


Figure 5 Ratio of α/β as a Function of Frequency for Commonly-Used Ferrite Materials

Calculating Real Constitutive Parameters from Complex Constitutive Parameters

Because some contemporary FDTD models accept only real constitutive parameters (ϵ , μ , and σ) as inputs, there may be cases when it is desirable to calculate real (i.e., non-complex) constitutive parameters given the complex ones defining a medium. That calculation is the topic of this section.

The calculation of real parameters from complex parameters is obtained through a frequency-domain analysis. Although applied and verified in FDTD, this approach can be used with other techniques as well. Real-valued parameters are selected so that they provide a close match with the electromagnetic properties of a medium (i.e., impedance and propagation constants) as calculated using the actual, complex-valued parameters. Calculations are performed using a one-dimensional model with normal field incidence as described below. The real constitutive parameters obtained from the complex values are

referred to as synthetic constitutive parameters in this development. The concept of applying synthetic parameters is not new to FDTD, as a synthetic conductivity has been used in the past to achieve closer agreement with the actual reflection coefficient for good conductors [12].

In cases where magnetic losses can be ignored ($\mu''=0$), the calculation of real constitutive parameters is readily determined by matching either the intrinsic impedance or the propagation constant:

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega(\varepsilon' - j\varepsilon'')}} = \sqrt{\frac{j\omega\mu_{synth}}{\sigma_{synth} + j\omega\varepsilon_{synth}}} \quad (11)$$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega(\varepsilon' - j\varepsilon''))} = \sqrt{j\omega\mu(\sigma_{synth} + j\omega\varepsilon_{synth})} \quad (12)$$

For this magnetically-lossless case, identical η and γ are obtained if $\sigma_{synth} = \sigma + \omega\varepsilon''$, $\varepsilon_{synth} = \varepsilon'$ and $\mu_{synth} = \mu$.

Unfortunately, this approach is not as straightforward for media with magnetic losses (e.g. ferrites), because there is no algebraic solution for matching media impedance. However, values of the synthetic constitutive parameters can be adjusted so that reflection from and transmission into the medium will be the same as when using the complex constitutive parameters, and that is the approach taken here. Further, it is assumed that there are no significant multiple reflections within the material that would change the surface impedance of the medium.

Reflection and transmission coefficients are calculated assuming normal field incidence from free space, using the standard formulae:

$$\Gamma = \frac{\eta - \eta_0}{\eta + \eta_0} \quad \text{and} \quad T = \frac{2\eta}{\eta + \eta_0},$$

The magnitudes of these coefficients are first calculated using the actual complex-valued constitutive parameters, and then they are matched using an intrinsic impedance calculated from the synthetic constitutive parameters:

$$\eta_{synth} = \sqrt{j\omega\mu_{synth} / (\sigma_{synth} + j\omega\varepsilon_{synth})}$$

Because of the transcendental nature of this solution, computer code was written to search for appropriate synthetic values. That algorithm searches through the three-dimensional parameter space (defined by μ , ε , and σ) until the transmission and reflection coefficient magnitudes are matched. Since three synthetic variables are used to represent four actual variables, there is no unique solution. The algorithm picks the one that most closely matches the magnitude of the impedance of the original medium as defined by its complex-valued constitutive parameters.

Validation of Synthetic Constitutive Parameters and Examples of Cell Sizes in Ferrites

To verify that the synthetic constitutive parameters identified above will produce accurate results when used in an FDTD model, a validation effort was performed. In that effort, normal incidence and reflection were modeled in FDTD using both the original complex-valued and synthetic constitutive parameters, and the results were compared against an exact frequency-domain solution.

Although validation was performed for several ferrites types, the results shown here is for the material “Anechoic Chamber”; the results obtained for other ferrite materials is very similar to the results shown here. The complex constitutive parameters for this ferrite material is given in Table 1 for the range of frequencies studied. The Δx given for a particular frequency was selected to give the equivalent finite-differencing error as would occur in a lossless medium sampled at 10 cells per wavelength, and it was determined using the approach outlined in the previous section.

Frequency [Hz]	μ_r'	μ_r''	ϵ_r'	ϵ_r''	Δx (m)
24950000	12.16434	0.126852	61.88421	259.2611	0.0198
54850000	12.33347	0.00381	18.00793	126.5468	0.0125
1.07E+08	12.35166	0.03079	6.616123	67.49062	0.0087
2.87E+08	12.39222	0.011837	1.025142	26.47257	0.005
4.06E+08	12.42664	0.012603	0.234895	18.80141	0.0041
6.75E+08	12.48441	0.009533	0.29848	11.14053	0.0032
1.00E+09	12.54819	0.009975	0.371113	7.294982	0.0027
1.30E+09	12.59211	0.042656	0.366589	5.481378	0.0024
1.60E+09	12.61644	0.077229	0.366909	4.34567	0.0022
2.01E+09	12.58664	0.142766	0.381753	3.337931	0.002
2.35E+09	12.52966	0.211981	0.376177	2.743077	0.0019
2.6E+09	12.47664	0.229623	0.361339	2.399201	0.0018
3E+09	12.45437	0.182025	0.304257	1.969054	0.0018

Table 1. Complex Constitutive Parameters and Corresponding FDTD Cell Size for the Ferrite Material “Beads & Chokes 1”

To calculate the reflection and transmission data, the FDTD models were run for each of the frequencies shown in Table 1 using the cell size (Δx) indicated in the table. A Gaussian pulse excitation was used, and a Fourier transform was employed to extract the reflection and transmission coefficients at the frequency being evaluated. Certainly, this is not an efficient way to use FDTD, but it was necessary to accomplish the validation.

Results of the reflection and transmission coefficient comparisons are shown in Figure 6, and these verify that the synthetic constitutive parameters do afford an accurate modeling of reflection and transmission. It is somewhat surprising that the synthetic parameters result in closer agreement to the exact reflection coefficient values than do the actual complex constitutive parameters. One likely explanation for this is that by lumping both electric and magnetic losses in only one of the field variables (the electric field in this case), the error will be lower than when accounting for them separately in the spatially

and temporally-offset electric and magnetic field variables.

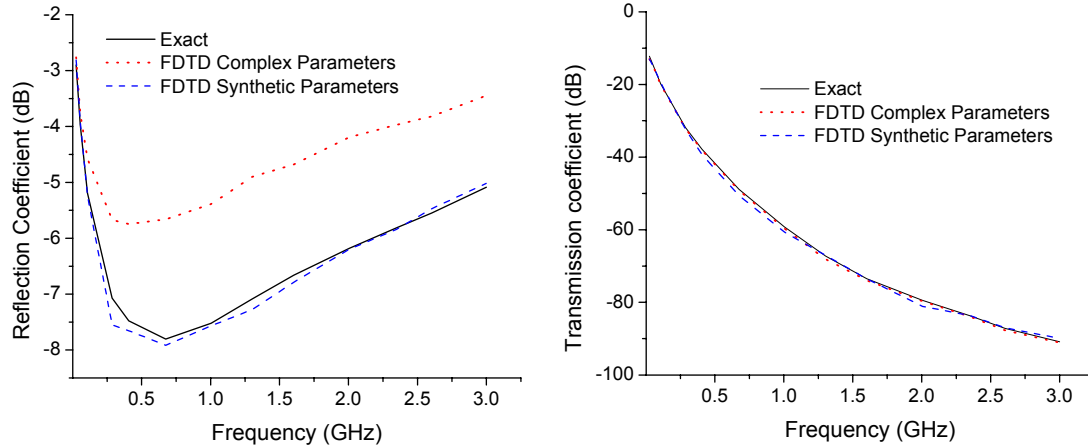


Figure 6 FDTD Modeled and Exact Reflection Coefficients

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